MATH 130 Review Notes

example 1 Solve for x

$$2x + 5 < 8$$

solution 1

$$2x + 5 < 8 \iff 2x < 3 \iff x < \frac{3}{2} \;\; or \;\; (-\infty, 3/2)$$

example 2 Solve for x

$$-4x \geq 20$$

solution 2

$$-4x \geq 20 \iff x \leq -5 \;\; or \;\; (-\infty, -5]$$

Notice the switch in the inequality caused by mult/div of a negative number

example 3 Simplify

$$\frac{|\,-12 + 4\,|}{|\,16 - 12\,|}$$

solution 3

$$\frac{|\,-12 + 4\,|}{|\,16 - 12\,|} = \frac{|\,-8\,|}{|\,4\,|} = \frac{8}{4} = 2$$

example 4 Simplify

$$27^{2/3}$$

solution 4

$$27^{2/3} \;=\; (\sqrt[3]{27})^2 \;=\; 3^2 \;=\; 9$$

example 5 Simplify

$$8^{-4/3}$$

solution 5

$$8^{-4/3} \;=\; \frac{1}{8^{4/3}} \;=\; \frac{1}{(\sqrt[3]{8})^4} \;=\; \frac{1}{2^4} \;=\; \frac{1}{16}$$

example 6 Simplify

$$(\sqrt{3})^0$$

solution 6

$$(\sqrt{3})^0 \;=\; 1$$

example 7 Is the following statement true or false?

$$(3^2)(2^2) \;=\; 6^2$$

solution 7

$$TRUE\ since\ \ (3^2)(2^2) \;=\; (9)(4) \;=\; 36 \;=\; 6^2$$

example 8 Is the following statement true or false?

$$3^3 + 3 \;=\; 3^4$$

solution 8

$$FALSE\ since\ \ 3^3 + 3 \;=\; 27 + 3 \;=\; 30 \;\neq\; 3^4$$

example 9 Rewrite using positive exponents only

$$(xy)^{-2}$$

solution 9

$$(xy)^{-2} \;=\; \frac{1}{(xy)^2}$$

example 10 Simplify

$$(x^2y^{-3})(x^{-5}y^3)$$

solution 10

$$(x^2y^{-3})(x^{-5}y^3) \;=\; \frac{x^2y^3}{y^3x^5} \;=\; \frac{1}{x^3} \;=\; x^{-3}$$

example 11 Simplify

$$\frac{5x^6y^3}{3x^2y^9}$$

solution 11

$$\frac{5x^6y^3}{3x^2y^9} \;=\; \frac{5x^4}{3y^6}$$

example 12 Simplify

$$\frac{x^{3/5}}{x^{-1/5}}$$

solution 12

$$\frac{x^{3/5}}{x^{-1/5}} \;=\; x^{3/5}x^{1/5} \;=\; x^{(3/5+1/5)} \;=\; x^{4/5}$$

example 13 Rationalize the denominator

$$\frac{3}{2\sqrt{x}}$$

solution 13

$$\frac{3}{2\sqrt{x}} \;=\; \left(\frac{3}{2\sqrt{x}}\right)\left(\frac{\sqrt{x}}{\sqrt{x}}\right) \;=\; \frac{3\sqrt{x}}{2x}$$

example 14 Rationalize the denominator

$$\frac{2y}{\sqrt{3y}}$$

solution 14

$$\frac{2y}{\sqrt{3y}} \;=\; \left(\frac{2y}{\sqrt{3y}}\right)\left(\frac{\sqrt{3y}}{\sqrt{3y}}\right) \;=\; \frac{2y\sqrt{3y}}{3y} \;=\; \frac{2\sqrt{3y}}{3}$$

example 15 Simplify

$$3(2a-b)-4(b-2a)$$

solution 15

$$3(2a-b)-4(b-2a) \;=\; 6a-3b-4b+8a \;=\; 14a-7b$$

example 16 Simplify

$$x-[2x-(-x-[1-x])]$$

solution 16

$$x-[2x-(-x-[1-x])] \;=\; x-[2x-(-x-1+x)] \;=\; x-[2x+x+1-x] \;=\; x-2x-x-1+x \;=\; -x-1$$

example 17 Simplify

$$(a+4)^2$$

solution 17

$$(a+4)^2 \;=\; (a+4)(a+4) \;=\; a^2+4a+4a+16 \;=\; a^2+8a+16$$

example 18 Factor as much as possible

$$4x^5 - 12x^4 - 6x^3$$

solution 18

$$4x^5 - 12x^4 - 6x^3 \;=\; 2x^3(2x^2 - 6x - 3)$$

example 19 Factor

$$4a^2 - b^2$$

solution 19

$$4a^2 - b^2 \;=\; (2a + b)(2a - b)$$

example 20 Find the roots by factoring

$$x^2 + x - 12$$

solution 20

$$x^2 + x - 12 \;=\; (x+4)(x-3) = 0 \iff (x+4) = 0 \;\; or \;\; (x-3) = 0 \iff x = -4 \;\; or \;\; x = 3$$

example 21 Find the roots by factoring

$$3x^2 - x - 4$$

solution 21

$$3x^2 - x - 4 \;=\; (3x-4)(x+1) = 0 \iff (3x-4) = 0 \;\; or \;\; (x+1) = 0 \iff x = 4/3 \;\; or \;\; x = -1$$

example 22 Simplify

$$\frac{x^2 + x - 2}{x^2 - 4}$$

solution 22

$$\frac{x^2 + x - 2}{x^2 - 4} \;=\; \frac{(x+2)(x-1)}{(x+2)(x-2)} \;=\; \frac{x-1}{x-2}$$

example 23 Subtract and then simplify

$$\frac{2x}{2x-1} - \frac{3x}{2x+5}$$

solution 23

$$\frac{2x}{2x-1} - \frac{3x}{2x+5} = \left(\frac{2x}{2x-1}\right)\left(\frac{2x+5}{2x+5}\right) - \left(\frac{3x}{2x+5}\right)\left(\frac{2x-1}{2x-1}\right)$$

$$= \frac{2x(2x+5)-3x(2x-1)}{(2x-1)(2x+5)} = \frac{4x^2+10x-6x^2+3x}{(2x-1)(2x+5)} = \frac{-2x^2+13x}{(2x-1)(2x+5)}$$

example 24 Divide and then simplify

$$\frac{2a^2-2b^2}{b-a} \div \frac{a^2+2ab+b^2}{4a+4b}$$

solution 24

$$\frac{2a^2-2b^2}{b-a} \div \frac{a^2+2ab+b^2}{4a+4b} = \frac{2a^2-2b^2}{b-a} \cdot \frac{4a+4b}{a^2+2ab+b^2}$$

$$= \frac{2(a+b)(a-b)}{b-a} \cdot \frac{4(a+b)}{(a+b)(a+b)} = \frac{8(a-b)}{b-a} = \frac{-8(b-a)}{b-a} = -8$$

example 25 Find the equation of the circle with radius 5 and center (2,-3)

solution 25 Since the general equation of a circle of radius r centered at (h, k) is:

$$(x-h)^2 + (y-k)^2 = r^2$$

$$\textit{The equation is} \quad (x-2)^2 + (y+3)^2 = 25$$

example 26 Find the equation of the circle with radius 2 and center (-3,-5)

solution 26

$$\textit{The equation is} \quad (x+3)^2 + (y+5)^2 = 4$$

example 27 Find the equation of the circle with radius 6 centered at the origin

solution 27

$$\textit{The equation is} \quad x^2 + y^2 = 36$$

example 28 Find the slope of the line that passes through (4,5) and (3,8).

solution 28

$$m = \frac{y_2 - y_1}{x_2 - x_1} \implies m = \frac{8-5}{3-4} = \frac{3}{-1} = -3$$

example 29 Find the equation of the line that passes through (2,4) and (3,7).

solution 29

$$m = \frac{7-4}{3-2} = \frac{3}{1} = 3 \implies \textit{the equation is } y-4 = 3(x-2) \textit{ or } y-7 = 3(x-3) \textit{ or } y = 3x-2$$

example 30 Find the equation of the line that passes through (-5,-4) and is parallel to the line joining (-3,2) and (6,8).

solution 30 So we need a line parallel to one with the following slope:

$$m = \frac{8-2}{6-(-3)} = \frac{6}{9} = \frac{2}{3}$$

Parallel means the same slope so we need a line through (-5,-4) with slope 2/3

$$\implies \textit{the equation is } y+4 = \frac{2}{3}(x+5) \textit{ or } y = \frac{2}{3}x - \frac{2}{3}$$

example 31 Find the equation of the line that passes through (-5,-4) and is perpendicular to the line joining (-3,2) and (6,8).

solution 31 So we need a line perpendicular to one with the following slope:

$$m = \frac{8-2}{6-(-3)} = \frac{6}{9} = \frac{2}{3}$$

Perpendicular means the slope is the negative reciprocal so we need a line through (-5,-4) with slope -3/2

$$\implies \textit{the equation is } y+4 = -\frac{3}{2}(x+5) \textit{ or } y = -\frac{3}{2}x - \frac{23}{2}$$